Mechanism Design With Money

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Motivation

Mechanism Design and Social Choice

Design rules in order to make decisions based on people preferences when their interests are conflicting.

Game Theory

Game Theory

Studies strategic situations where players choose different actions in an attempt to maximize their returns.

Outcome Prediction - Solution Concepts

- Nash Equilibrium
- Pure Nash Equilibrium
- Dominant Strategy

Mechanism Design

Mechanism Design

Mechanism design is the art of designing rules of a game to achieve a specific outcome under a certain solution concept.

Social Choice as a Game

- A set A of different alternatives
- A set of n voters (the agents) N
- Each agent i has a linear order $\succ_i \in L$ over the set A

A function (mechanism) $f : L^n \rightarrow A$ that maps the agents' preferences to a single alternative is called social choice function.

Properties

Onto

$\forall a \in A, \exists x \in L^n$ such that $f(x) = a$

Unanimous

if $\exists a \in A$ such that $\forall b \in A$ and $i \in N$, $a \succ_i b$ then $f(\succ_1, \ldots, \succ_n) = a$

Pareto Optimal

if $f(\succ_1, \ldots, \succ_n) = a$, then $\nexists b \in A$ such that $b \succ_i a, \forall i \in N$

Properties - Incentive Compatibility

Strategic Manipulation by agent i

$\exists \succ_1, \ldots, \succ_n, \succ_i' \in L$ such that $b \succ_i a$ where $a = f(\succ_1, \ldots, \succ_i, \ldots, \succ_n)$ and $b = f(\succ_1, \ldots, \succ_i', \ldots, \succ_n)$.

Strategyproofness

A social choice function is called **incentive compatible** or **strategyproof** or **truthful** if no agent can strategically manipulate it.

Impossibility Result

Gibbard-Satterthwaite

Let f be an incentive compatible social choice function onto A, where $|A| \geq 3$, then f is a dictatorship.

Escape Routes

- Money
- Randomization
- Restricted domain of preferences

Setting and Outline

Measuring Preferences with Money

Each agent has a value for every alternative $a \in A$, which is given by a private function $v_i : A \to \mathbb{R}$ where $v_i \in V_i$.
The value $v_i(a)$ corresponds to the amount of money agent i is willing to pay in order to force the outcome a.

Extending the notion of a mechanism

A mechanism is a social choice function $f : V_1 \times V_2 \times \cdots \times V_n$ and a vector of payment functions $p_1, p_2, \ldots, p_n$, where $p_i : V_i \to \mathbb{R}$ is the amount that player i pays.

Desirable Properties

Incentive Compatibility

Mechanism $(f, p_1, ..., p_n)$ is **incentive compatible** if for each player i, every $v_1, ..., v_n$ and every v_i', we have that
$v_i(f(v_i, \vec{v}_{-i})) - p_i(v_i, \vec{v}_{-i}) \geq v_i(f(v_i', \vec{v}_{-i})) - p_i(v_i', \vec{v}_{-i})$

Individual Rationality

Mechanism $(f, p_1, \ldots, p_n)$ is **individually rational** if for each player i and every $v_1, ..., v_n$, we have that $v_i(f(v_1, ..., v_n)) - p_i(v_i, ..., v_n)) \geq 0$

No Positive Transfers

Mechanism $(f, p_1, ..., p_n)$ has **no positive transfers** if for each player i and every $v_1, ..., v_n$, we have that $p_i(v_i, ..., v_n)) \geq 0$

Single Item Auction

Selling a single item

The set of alternatives is the set of possible winners
$A = \{i - wins | i \in N\}$
The agent valuations are $v_i(i - wins) = w_i$ and $v_i(j - wins) = 0 \forall j \neq i$.
The agent with the highest w_i should get the item.

How to choose payments

- **No payments** Player i should increase his bid to get the item.
- **Winner pays bid** In the case where the other agent valuations are lower the winner should decrease his bid

Not incentive compatible

Single Item Auction - Truthful Mechanisms

Pay all agents

Give the item to the highest bidder and pay everyone else the value of the winning bid.
Truthful but not efficient (positive transfers)

Winner pays second highest bid - Vickrey Auction

Give the item to the highest bidder i and make him pay $p^* = \max_{j \neq i} w_j$.
Everyone else doesn't pay anything.
Truthful and efficient (positive transfers)

General Setting

Maximizing Social Welfare

There exists payment functions such that the mechanism that maximizes the social welfare i.e. $\sum_{i \in N} v_i(a)$ is incentive compatible.

Vickrey-Clarke-Groves Mechanisms

- $f(v_1, ..., v_n) \in argmax_{a \in A} \sum_{i \in N} v_i(a)$
- $p_i(v_i, \vec{v}_{-i}) = h_i(\vec{v}_{-i}) - \sum_{j \neq i} v_j(f(v_1, ..., v_n))$

where $h_i \in V_{-i} \to \mathbb{R}$ arbitrary functions

Theorem

Every VCG mechanism is incentive compatible.

Choosing the h_i's

$h_i = 0$

Extremely inefficient. The mechanism pays every player. The no positive transfers property is violated.

Clarke Pivot Rule

Choose each $h_i(\vec{v}_{-i}) = max_{b \in A} \sum_{j \neq i} v_j(b)$. (=max social welfare when player i doesn't participate.)
Player i is charged with the amount that the social welfare of the others decreased due to his choice.

Theorem

A VCG mechanism with the CPR makes no positive transfers.

Example

Summary & Example

	a	b	c	price	utility
I	1	5	10	5	10-5=5
II	7	6	4	0	4
III	9	8	7	0	7
sum	17	19	21		

What if player I lies and says that her value for c is 7?

	a	b	c	price	utility
I	1	5	~~10~~ 7	2	5-2=3
II	7	6	4		
III	9	8	7		
sum	17	19	18		

smaller than her utility when telling the truth!

Weighted VCG

Affine Maximizer

A social choice function f is called affine maximizer if for some player weights $w_1, ..., w_n \in \mathbb{R}$ and some weights $c_a \in \mathbb{R}$ for all $a \in A$, we have that $f(v_1, ..., v_n) = argmax_{a \in A}(c_a + \sum_{i \in N} w_i v_i(a))$

Weighted VCG

- $f(v_1, ..., v_n) = argmax_{a \in A}(c_a + \sum_{i \in N} w_i v_i(a))$
- $p_i(v_i, \vec{v}_{-i}) = h_i(\vec{v}_{-i}) - \sum_{j \neq i}(w_j/w_i)v_j(f(v_1, ..., v_n)) - c_a/w_i$

where $h_i \in V_{-i} \to \mathbb{R}$ arbitrary functions

Theorem (Roberts)

If $|A| \geq 3$, f is onto A, $V_i = \mathbb{R}^{|A|}$, then all incentive compatible mechanisms are Weighted VCG.

Setting and outline

Problem Statement

- A set of m items M to be divided among the agents ($A = (N \cup \{e\})^m$)
- For each agent i, $v_i : 2^M \to \mathbb{R}$ (monotone valuation function)
- Objective: Maximize social welfare

Difficulties

- The allocation problem is NP-complete to compute optimally
- How to retrieve and represent agent valuations? ($|A|$ is exponential)

Setting and outline

Avoiding the difficulties

- Focus on simpler cases of valuation functions (linear, single minded)
- Introduce approximation

VCG mechanisms

VCG mechanisms require the computation of the optimal social welfare. They don't work for approximations.

Single-Minded Case

Each agent i is interested in obtaining a certain bundle of items S_i.
$v_i(T) = v_i^*, \forall T \supseteq S_i$
$v_i(T) = 0, \forall T \subset S_i$

Proposition

The allocation problem among single-minded bidders is NP-hard.

We show this by a reduction to the INDEPENDENT-SET problem. The items are the edges and the bidders are the vertices. Each bidder's bundle is the set of his adjacent edges with value $v_i^* = 1$.

Theorem

No approximate mechanism exists with approximation ratio $m^{1/2-\epsilon}$.

Single-Minded Case

Greedy Mechanism

- Order the agents bids by $v_i^* / \sqrt{S_i}$ decreasingly
- Process agents in order and give them their desired bundle if available

Theorem

The greedy mechanism is incentive compatible and $\sqrt{m}$-approximate

The End

Thank you!